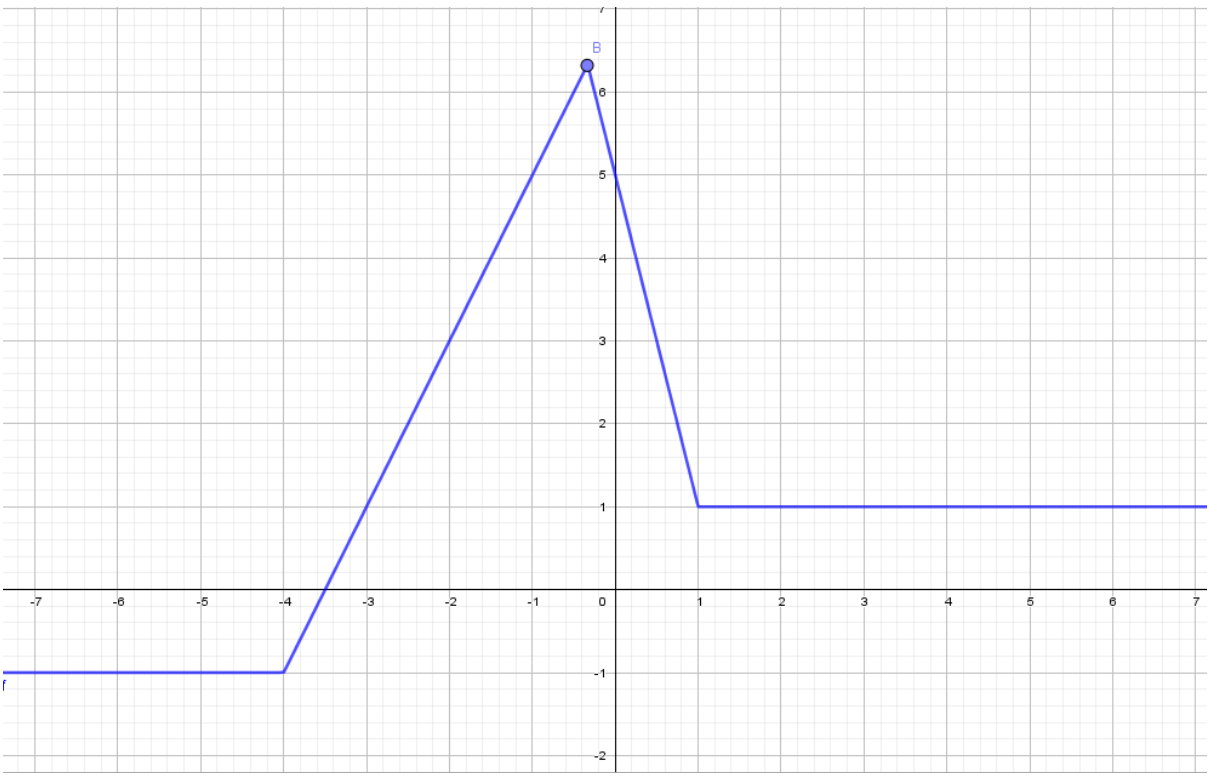

Point B is (-1/3, 19/3). Find the equation of this function.

Solution:

We know that we can only add/ subtract absolute values of linear functions to get this graph.

The equation should be in form of $|ax + b|$ added with other absolute linear functions.

The graph above has 3 corners. A rule about add absolute linear functions is that the number of corners in the graph is the same as the number of absolute value expression.

For example, $|ax + b| \pm |cx + d|$ will have 2 corners. As we need 3 corners, we know that we are going to need 3 expressions. $|ax + b| \pm |cx + d| \pm |ex + f|$

From the graph we know that the function is constant at large or small x values. That means the x needs to cancel out at large or small values. This leaves us with $a \pm c \pm e = 0$, that gives us 2 cases:

$$a - c - e = 0$$

$$a + c - e = 0$$

All the signs cannot be plus as that would not make the x cancel out.

Let's analyse both cases.

Case 1: $f(x) = |ax + b| + |cx + d| - |(a + c)x + f|$

For large X positives, this simplifies to $f(x) = b + d - f$

And to match the graph $b + d - f = 1$ => $f = b + d - 1$

Case 2: $f(x) = |ax + b| - |cx + d| - |(a + c)x + f|$

If we do the same thing as Case 1 then we see here: $f = b - d - 1$

We can put the graphs of desmos with random positive values for the constants and see which case matches out graph more. https://www.desmos.com/calculator/6brxksb5vc

Here the blue graph is case 1 and we see it has an upward pointing corner and that's what we need.

Therefore the equations will be: $f(x) = |ax + b| - |cx + d| - |(a + c)x + (b + d - 1)|$

So now we have filtered out which signs to use. But now we need to find the corners at x values

-4, -1/3 and 1.

To get a corner at x equals to some constant. The absolute expression should be equal to 0. Meaning

$$ax + b = 0, when\ x = -4$$

$$cx + b = 0, when\ x = -1$$

$$(a + c)x + (b + d - 1) = 0, when\ x = -1/3$$

This gives us a system of equation:

$$\begin{cases} a(-4) + b = 0 \\ c(-1) + b = 0 \\ -\frac{1}{3}(a + c) + b = d - 1 = 0 \end{cases}$$

With some simple substitution we can determine that $b = 4a$ and $d = -c$

We know that $b - 4a = 0$ and that $b = 4a$ This leaves us with only once solution: $a = 1\,; b = 4$

Let's substitute this in the equation:

$$f(x) = |1x + 4| - |cx + d| - |(a + c)x + (b + d - 1)|$$

We also know that $d = -c$. Let's substitute this also in the equation.

$$f(x) = |1x + 4| - |cx + (-c)| - |(a + c)x + (b + (-c) - 1)|$$

That leaves us with only one constant to find. We can plug this equation in desmos and add a slider for c. We need the function to pass through point B. Using trail and error c = 2.

And the function for the graph is:

$$f(x) = |1x + 4| - |cx + (-2)| - |(a + 2)x + (b + (-2) - 1)|$$

In desmos: https://www.desmos.com/calculator/oz9ukfivxx